**Control of the Double Inverted Pendulum**

**on Cart**

**Mechatronics - MECH 6741**

**Course Project**

**Professor**

Dr. Chun-Yi Su

**By**

|  |  |  |
| --- | --- | --- |
| Amir Sayadi | Shervin Foroughi | Saud M. Altamimi |
| Student ID# 40106366 | Student ID# 40038869 | Student ID# 40059341 |

**Winter 2019**

**Table of Contents**

[1 Introduction 1](#_Toc5893633)

[2 Mathematical modeling of double inverted pendulum 1](#_Toc5893634)

[3 System identification 5](#_Toc5893635)

[3.1 DIP setup and specification 5](#_Toc5893636)

[3.1.1 The linear motion servo plant (Cart) 6](#_Toc5893637)

[3.1.2 Power Module 7](#_Toc5893638)

[3.1.3 The I/O board 7](#_Toc5893639)

[3.1.4 Pendulums 8](#_Toc5893640)

[3.1.5 Encoders 8](#_Toc5893641)

[3.2 Identification approach 8](#_Toc5893642)

[4 Verification of the results 11](#_Toc5893643)

[5 Control Design 13](#_Toc5893644)

[5.1 History 13](#_Toc5893645)

[5.2 Overview of DIP system control strategies 14](#_Toc5893646)

[5.3 Swing-up control 14](#_Toc5893647)

[5.4 Closed loop stabilization control 15](#_Toc5893648)

[6 Experimental Results 18](#_Toc5893649)

[7 Conclusion 19](#_Toc5893650)

[8 Contributions 19](#_Toc5893651)

[9 References 20](#_Toc5893652)

Appendix A 21

**List of Figures**

[Figure 1: Schematic of the Double Inverted Pendulum on a Cart 2](#_Toc5893706)

[Figure 2: Double inverted pendulum setup 6](#_Toc5893707)

[Figure 3: Cart with servomotor drive 6](#_Toc5893708)

[Figure 4: Power module 7](#_Toc5893709)

[Figure 5: I/O board 7](#_Toc5893710)

[Figure 6: DIP system encoders 8](#_Toc5893711)

[Figure 7: Open-loop input signals 10](#_Toc5893712)

[Figure 8: Block diagram of MATLAB Simulink model for calculating velocities of the DIP system 10](#_Toc5893713)

[Figure 9: Comparison between simulated and measured values of state parameters , , 12](#_Toc5893714)

[Figure 10: Comparison between simulated and measured values of state parameters , , 12](#_Toc5893715)

[Figure 11: Case studies for LQR performance 16](#_Toc5893716)

[Figure 12: Extracted results during the LQR controller operation for 10 seconds 17](#_Toc5893717)

[Figure 13: Results for system responses and with respect to applied step function 17](#_Toc5893718)

[Figure 14: Block diagram of MATLAB Simulink model for DIP control system 18](#_Toc5893719)

[Figure 15: The inserted pins for limiting the rotation of second pendulum 19](#_Toc5893720)

# Introduction

Double inverted pendulum (DIP) on a cart is one of the nonlinear control problems which can be used for system-based experiments in order to test the linear and nonlinear control laws such as stabilization of the dynamic systems [1, 2]. Nonlinearity, being multivariable and fast reaction as well as instability are the essential characteristics of this system [2]. The instability and nonlinearity of this system makes it the perfect model for examining the various control algorithms such as PID controls, LQR controls, neural network, fuzzy control, etc. [2]. Commonly, there are two different this system, double inverted pendulum-cart system and the rotary double inverted pendulum. All these systems consist of two inverted pendulums that are assembled on a cart or servomotor as a motion generators and rotate about their pivot points. Pendulums are controlled and stabilized by applying external force or torque to the motion generators. The double pendulum system consists of one input which is the cart excitation force and three outputs: angular position of rods and linear position of the cart.

The objectives of this project are (i) formulation of dynamic model of DIP system (ii) identification of constant parameters of the dynamic model (iii) verification of obtained state parameters from simulation and real system measurement (iv) swing up the pendulum rods with utilizing the control function and (v) stabilizing the rods at their unstable equilibrium point. However, choosing the appropriate control algorithm and developing the system based on that are the final goal of this project.

# Mathematical modeling of double inverted pendulum

In this section the mathematical modeling of the DIP will be performed in order to provide a precise description of the system. This model possess sets of nonlinear equations of system motions including the constant parameters of the system. The model simulation is performed by utilizing the MATLAB Simulink software. Table 1 shows the DIP system constants’ notations and units.

|  |  |  |
| --- | --- | --- |
| Table 1 Notation and units of constant parameters of the DIP system | | |
| Parameter | Unit | Description |
| mc | [Kg] | Mass of the cart |
| m1, m2 | [Kg] | Mass of the pendulums |
| L1, L2 | [m] | Length of the pendulums |
| l1, l2 | [m] | Distance from pivot joints to the pendulums’ center of mass |
| I1, I2 | [Kg m2] | Pendulums’ moment of Inertia |
| g | m/s2 | Gravity constant |
|  | [m] | Cart position |
|  | [rad] | Pendulums’ angles |

Figure 1 displays the schematic of DIP system on a cart which will be studied in this project.

|  |
| --- |
|  |
| Figure 1: Schematic of the Double Inverted Pendulum on a Cart |

In this study the desired equations of motion are found by implementing Lagrange’s equations:

|  |  |
| --- | --- |
|  | ( ‎2‑1 ) |

where the (*i* =1, 2, 3,….., r) is a set of generalized coordinates, denotes generalized forces or moments acting in direction of coordinates *q* and *L* is the lagrangian function defined as:

|  |  |
| --- | --- |
|  | ( ‎2‑2 ) |

in which *T* is the kinetic energy and *V* presents the potential energy. For the presented DIP system kinetic energy *T* and potential energy *P* are obtained by summation of individual components’ energies. For the system shown in Figure 1 the independent coordinate parameters are defined as and . Therefore, by considering the individual external force u exerted to the system along x coordinate and neglecting the friction of the system, equation (2-1) can be expanded as follow:

|  |  |
| --- | --- |
|  | ( ‎2‑3 ) |
|  | ( ‎2‑4) |
|  | ( ‎2‑5) |

Since the system has three structural elements (cart and two pendulum rods), system energies are defined as follow:

|  |  |
| --- | --- |
|  | ( ‎2‑6 ) |
|  | ( ‎2‑7) |

In which indices *c*, 1 and 2 denote the cart, 1st and 2nd pendulum rods respectively. Assuming the components of system, energy is calculated as follow:

|  |  |
| --- | --- |
|  | ( ‎2‑8 ) |
|  | ( ‎2‑9) |
|  | ( ‎2‑10 ) |
|  | ( ‎2‑11 ) |
|  | ( ‎2‑12 ) |
|  | ( ‎2‑13 ) |

Substituting energy equations in equation 2-2, the Lagrangian function of the system becomes:

|  |  |
| --- | --- |
|  | ( ‎2‑14 ) |

Using the obtained result for *L,* equations 2-3, 2-4 and 2-5 can be rewritten as follow:

|  |  |
| --- | --- |
|  | ( ‎2‑15 ) |
|  | ( ‎2‑16) |
|  | ( ‎2‑17) |

These equations are nonlinear due to the presence of the general terms , ,. The equation 2-18 presents the compact matrix form of the last three equations:

|  |  |
| --- | --- |
|  | ( ‎2‑18 ) |

where:

|  |  |
| --- | --- |
|  | ( ‎2‑19 ) |
|  | ( ‎2‑20 ) |
|  | ( ‎2‑21 ) |
|  | ( ‎2‑22 ) |
|  | ( ‎2‑23 ) |

Since in Figure 1 it is assumed that the centers of mass of the rods locate at the geometrical center, for both rods “” and “” will be “” and “” respectively. As a result, by substituting magnitudes of “” and “” parameters will be defined as:

|  |  |  |
| --- | --- | --- |
|  | (a) | ( ‎2‑24 ) |
|  | (b) |
|  | (c) |
|  | (d) |
|  | (e) |
|  | (f) |
|  | (g) |
|  | (h) |

So, considering the equation 2-18 and multiplying it by the matrix as well as defining the state variables and substituting in this equation, the nonlinear form of state equation will be obtained as below:

|  |  |  |
| --- | --- | --- |
| = | (a) | ( ‎2‑25 ) |
|  | (b) |
|  | (c) |
|  | (d) |
|  | (e) |
|  | (f) |
|  | (g) |
|  | (h) |

The equation 2-26 (a) could be written in a following compact form as well [1]:

|  |  |
| --- | --- |
|  | ( ‎2‑26 ) |

So far, the dynamic model of the DIP system as well as nonlinear form of state equation have been derived by implementing the Lagrangian function. In next section the identification of this system will be performed in order to find the parameters presented in equation 2-24.

# System identification

In order to identify the unknown parameters presented in equations 2-24 (, the on-line system identification will be performed by utilizing the energy theorem. As a result, it is needed to form equations that can be solved for . In this approach the on-line data needs to be collected from the real system. In the following, the DIP setup used in this project is introduced in detail.

## DIP setup and specification

This section introduces the hardware and system components used in order to successfully erect and balance the pendulum. Figure 2 illustrates the system setup used in this project.

|  |  |
| --- | --- |
| Power supply Module  Cart  Rack | Pendulums  2nd pendulum encoder |
| (a) | (b) |
| Figure 2: Double inverted pendulum setup | |

The main parts of this setup are linear motion servo plant (Cart), Power supply module, I/O board, Pendulums and encoders that are introduced. Some of detail information are extracted from reference [3].

### The linear motion servo plant (Cart)

Figure 3 displays the cart of the DIP system. The servomotor connects to the power supply module and provides the cart motion. The input voltage limitation of the servomotor is ±6 volts. The rack and pinion mechanism of the system converts the rotary motion of the motor to the linear motion.

|  |
| --- |
| Pinion connected to Cart motor axis  Position pinion connected to Cart encoder  Pendulum axis connected to 1st pendulum encoder  Rack  1st pendulum connector |
| Figure 3: Cart with servomotor drive |

### Power Module

Figure 4 displays the power module with a linear power operational amplifier along with a dual output DC power supply set.

|  |
| --- |
| To motor |
| Figure 4: Power module |

The DC power supply has three ports +12V,-12V and GND, which supply the power to the components. The operational amplifier has found binding posts, it can be used in standard op-amp configurations. Its two inputs are labeled (-) and (+) for the inverting input and non-inverting inputs respectively.

### The I/O board

Figure 5 shows the Q4 board which has multiple I/O ports and supports digital and analog sensors and is responsible for data acquisition. It provides four quadrature encoder inputs, used for transferring the encoders of the cart and pendulums data to the computer unit.

|  |
| --- |
| Signal port of cart’s encoder  Signal port of 1st pendulum’s encoder  Signal port of 2nd pendulum’s encoder |
| Figure 5: I/O board |

### Pendulums

As it was shown in Figure 2 the DIP system includes two pendulums. The socket joint connects the system of jointed pendulums to the cart’s axis. These pendulums are free to rotate about their joints due to the cart motion.

### Encoders

The DIP system include three optical encoders. The cart and pendulums’ positions are measured with these three sensors. Using the encoder provides the possibility for sensing the wide range of pendulum angular positions up to inverted position [3]. The cart’s linear position encoder performs the measurement through the rack-pinion system. Figure 6 displays these encoders assembled in the system.

|  |  |
| --- | --- |
| Cart’s encoder  1st pendulum’s encoder | 2nd pendulum’s encoder |
| a) Cart’s and 1st pendulum encoders | b) Encoder for 2nd pendulum |
| Figure 6: DIP system encoders | |

## Identification approach

According to the energy theorem, the work done by the system in a certain period is equal to the total energy change of the system. By neglecting the effect of the friction, in the DIP mechanism, the motor performs the work and the energy change of the system is equal to the summation of energy change for each individual elements, two pendulums and the cart. The equation 3-1 displays the energy theorem:

|  |  |
| --- | --- |
|  | ( ‎3‑1 ) |

which the is the work done by motor, , and are kinetic energy, potential energy and total energy of the system respectively at the times and . For the DIP system, by using equations (2-8 … 2-13) and substituting the unknown parameters, the total energy equation of the system at the time is obtained as:

|  |  |
| --- | --- |
|  | ( ‎3‑2 ) |

This equation is linear in parameters . The work done by the motor between two times  and is determined as below:

|  |  |
| --- | --- |
|  | ( ‎3‑3 ) |

in which is the motor constant that is considered 0.6, *V* is the supplied voltage to the motor and is the velocity of the cart.

Next, by utilizing the developed MATLAB Simulink model and identification code following steps will be applied:

* Driving the real system setup from fully extended downward resting position, with 2 open-loop input signals, 1 and 1.25 volts, as shown in Figure 7. Each of input signals applied for duration of 15s
* Recording the displacement parameters , , reported by the setup’s encoders at certain time steps () during driving the system. The time steps have been considered equal to 0.05s
* Calculating the velocities , and as well as energy of the system by implementing stored data at each time step ()
* Calculation of servomotor’s work, by substituting the in equation 3-3 for all successive time intervals

|  |
| --- |
|  |
| Figure 7: Open-loop input signals |

Figure 8 displays the block diagram of the MATLAB Simulink model.

|  |
| --- |
| D:\Uni\Mechatronics\Project\identification\model.png |
| Figure 8: Block diagram of MATLAB Simulink model for calculating velocities of the DIP system |

Finally, rewriting the equation 3-1 for all time intervals and substituting the recorded and calculated data from previous mentioned steps, results in forming number of linear equations with 8 unknowns (. In order to identify the optimized solution for these unknowns, the least square minimization method has been implemented in another developed MATLAB code, see appendix A. Table 2 shows the obtained results from the optimization algorithm.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Table 2 Identified unknowns from optimization algorithm | | | | | | | |
|  |  |  |  |  |  |  |  |
| 1.2432 | 0.0601 | 0.0489 | 0.0012 | 0.0071 | 0.0039 | 0.3216 | 0.2910 |

# Verification of the results

In order to provide the possibility of solving nonlinear system dynamic equation 2-26 numerically, verifying the results and comparing those with the measured data, this equation should be linearized by considering following assumptions:

* For stabilizing the pendulums along the vertical line, the angles and must be kept small which leads to: and .
* The and will be kept small, so the and will be negligible.

Considering the above assumptions, the state space equation is modified to following form:

|  |  |
| --- | --- |
|  | ( ‎4‑1 ) |

where:

|  |  |
| --- | --- |
|  | ( ‎4‑2 ) |
|  | ( ‎4‑3 ) |

By substituting the identified unknown parameters from Table 2 into the system dynamic equations, 4-1, 2 and 3, the equation 4-1 is solved numerically with RK45 method in MATLAB for calculating the , , , , , . On the other hand, the responses of the system to the input signals, which were introduced in Figure 2, have been measured and recorded in order to compare with the last numerically identified results. Figure 4 and 5 illustrate the comparison between the simulated and measured values of state parameters , , , , , during the 30 seconds.

|  |
| --- |
|  |
| Figure 9: Comparison between simulated and measured values of state parameters , , |

|  |
| --- |
|  |
| Figure 10: Comparison between simulated and measured values of state parameters , , |

Results show that the responses behavior of identified model is in good agreement with measured responses from driving the on-line DIP system. The difference between the magnitude of responses are due to the neglecting of friction in simulation model, error of data capturing by the encoders and linearizing of the system dynamic model. In order to identify the error of each state parameters , , , , , resulted from simulation with respect to one measured from real system the standard error has been calculated. The implemented equation of standard error is presented in equation 4-4.

|  |  |
| --- | --- |
|  | ( ‎4‑4 ) |

where N is the number of recorded samples, and are the measured and simulated state parameters respectively.

Table 3 displays the calculated values of the standard errors.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Table 3 Standard deviation of state parameters | | | | | |
| (m) | (rad) | (rad) | (m/s) | (rad/s) | (rad/s) |
| 0.0063 | 0.0035 | 0.0489 | 0.0610 | 0.0344 | 0.0839 |

As it has been mentioned so far, these errors obtained due to neglecting of the friction in simulation model, accuracy in data capturing by the encoders and linearizing of the system dynamic model.

So far, formulation of dynamic model of DIP system, identification of constant parameters of the dynamic model as well as verification of obtained state parameters from simulation and real system measurement have been performed. In following sections, the swing up and stabilization control systems for DIP will be designed, utilized and verified in order to identify the accuracy of them.

# Control Design

## History

There are number of different controllers, which have been developed in order to achieve the desired response and performance of the systems. The Proportional-Integral-Derivative (PID) control is a loop feedback controller that still is used in industrial fields and despite of its simplicity it provides the efficient solution to simple problems. But, in case of occurrence of complexity in the system such as nonlinearity it cannot work effectively. This weakness could be solved by combination of PID with other type of controllers such as adaptive, fuzzy, LQR, etc.

With the improvements made in the computing hardware and advent of efficient algorithms, the usage of optimal control has facilitated [4]. This control deals with optimizing a certain criteria for finding the control law for a system [5]. The linear quadratic regulator (LQR) and optimal neural network control are examples of this control system.

Adaptive control is another control method. In this method, the system repeatedly updates the controller parameters in order to minimize the error between actual system behavior and the model response [6].

The last two algorithms, LQR and Adaptive control, are the control methods which will be used in this research in order to control the performance of DIP on a cart system dynamics.

## Overview of DIP system control strategies

The control design of the DIP system is performed in two steps. First, designing the controller for the swing-up, next controller design for the stabilization of pendulums in an upright position by implementing MATLAB Simulink model. During this process of control, the swing-up controller is supposed to bring up the pendulums upright close to the stability point. Once the angular position of the pendulums reaches the certain predefined range of angles the system control will be switched to the closed loop stabilization controller that is supposed to take care of the system stabilization and maintains the DIP at the equilibrium position.

## Swing-up control

In this step, the energy method has been performed to derive an energy based swing-up controller for the DIP system. The energy equation of the DIP is defined as equation 5-1:

|  |  |
| --- | --- |
|  | ( ‎5‑1 ) |

Rewriting the energy equation to the matrix form, the equation 5-2 is obtained:

|  |  |
| --- | --- |
|  | ( ‎5‑2 ) |

Using the dynamic equation (2-18) and taking first derivative of , the will become:

|  |  |
| --- | --- |
|  | ( ‎5‑3 ) |

Defining the following Lyapunov function candidate:

|  |  |
| --- | --- |
|  | ( ‎5‑4 ) |

in which the is a potential energy at upright equilibrium point where and and is equal to *E*.

|  |  |
| --- | --- |
|  | ( ‎5‑5 ) |

So, *V* at stabilization point will be equal to zero and is defined as

|  |  |
| --- | --- |
|  | ( ‎5‑6 ) |

if the second term in right hand of above equation be chosen such that

|  |  |
| --- | --- |
|  | ( ‎5‑7 ) |

Where is a positive constant, then

|  |  |
| --- | --- |
|  | ( ‎5‑8 ) |

which means that the stability condition is satisfied. The control force is applied to the system by a DC-motor mounted on the cart.

## Closed loop stabilization control

In order to stabilize pendulums in the upright position the linear quadratic regulator (LQR) design is performed to design the proper controller. The LQR design will effectively return the state feedback gains needed to ensure stability of the system. However, to minimize the error of the cart position, a tracking controller is added by integration of cart position error with respect to the setting point over time. The gain adjustment of the integration result allows control over the zero steady state error convergence time. Since the LQR is applicable for the linear system dynamics, the linearized form of equation 2-26 for the DIP system around , has been introduced in equation 4-1, will be utilized to derive an approximate linear solution to the optimal control problem. The linearized equation is recalled again in following.

|  |  |
| --- | --- |
|  |  |

where:

|  |  |
| --- | --- |
|  |  |
|  |  |

Next, the quadratic performance index function which should be minimized by choosing the proper control signal , is defined as follow:

|  |  |
| --- | --- |
|  | ( ‎5‑9 ) |

where Q and R are positive definite matrices. In case of applying disturbance to the system or offset from the equilibrium point , the control signal should maintain the equilibrium state of the system at while the J is minimal. The input signal is defined as following:

|  |  |
| --- | --- |
|  | ( ‎5‑10 ) |

where P(t) is the solution of Riccati equation and K is the linear optimal feedback matrix.

In order to prove the successful performance of the LQR controller, Figure 11 shows the results extracted from Simulink model for three different cases for angle and .

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  | | |
|  |  |  |
|  |  |  |
|  | | |
|  |  |  |
|  |  |  |
|  | | |
| Figure 11: Case studies for LQR performance | | |

The graphs show that the accurate performance of the LQR around the upright position for angles in the interval [0 10].

In another case study, the pendulums were brought up manually and were left close the stabilization point at upright position. The response of the LQR controller during the stabilization of the DIP have been recorded for 10 seconds. Figure 12 shows the results.

|  |  |
| --- | --- |
|  |  |
|  |  |
| Figure 12: Extracted results during the LQR controller operation for 10 seconds | |

Figure 13 illustrates the system responses and to the applied step function about the stabilization point.

|  |  |  |
| --- | --- | --- |
|  |  |  |
| Applied step function |  |  |
| Figure 13: Results for system responses and with respect to applied step function | | |

Results shows a well performance of the LQR in order to maintain the stability of the system around the equilibrium point.

# Experimental Results

In order to ensure effective performance of the designed controllers, the integrated MATLAB simulink model, included both controllers, was developed, as shown in Figure 14.

|  |
| --- |
|  |
| Figure 14: Block diagram of MATLAB Simulink model for DIP control system |

Then, the DIP system on a cart was run with the cart’s motor input power 6 volts. The total cart movement on a rack was limited to 70 centimeters based on the total length of the rack. Since the possibility of working with each controllers, integrated or individually, have been considered in the structure of Simulink model, first, two controllers were examined individually which showed that both of them were working well. Next, the coupled controllers were tested. In this test, The swing-up happened successfully and the controller brought up the pendulums until the predefined angle 20◦, but the LQR was not able to stabilize the system at upright position and failed. By studying and performing multiple experiments it was observed that, whenever the control of the system switched to the LQR, the relative angle between second pendulum and the first rod was large. Therefore, due to the angular momentum of the second pendulum near the equilibrium point, the LQR controller was not able to reduce the large relative angle, so the system passed the stability point and kept rotating. This issue could happen because of the fabricated structural restriction against the rotation of second pendulum, which was made by inserting two screws at both sides of the second rod’s root, as shown in Figure 15, which prevents the proper balance of pendulums angular energy during the swing-up process.

|  |  |
| --- | --- |
| Screws | Screws |
| Figure 15: The inserted pins for limiting the rotation of second pendulum | |

In fact, the existing limitation in rotation of second pendulum imposes an unexpected nonlinearity in the linearized system, which causes to instability of the system.

# Conclusion

In this research, the control system for manipulation and stabilization of double inverted pendulum on a cart was designed and tested. The integrated controller included the swing-up and LQR controllers. Results from experiments showed the accurate and acceptable performance of each controllers. In case of using both controllers to perform the swing-up and stabilization continuously, the LQR was not able to maintain and stabilized the system at the upright position due to the structural restriction applied for rotation of second pendulum. To overcome this issue removing the motion limitation or using more than one controller for the swing-up in order to compensate the imposed disturbance to the system during swinging from the structural limitation are the tasks that can be proposed as future works.

# Contributions

Amir, Shervin and Saud put efforts on this project equally. All project tasks were defined at the beginning and distributed between them. Following table shows the tasks distribution.

|  |  |  |
| --- | --- | --- |
| **Task** | **Responsible person/s** | **Helped by** |
| Literature survey | All |  |
| Simulink Model | Amir | Shervin and Saud |
| Report | Shervin | Amir and Saud |
| Presentation | Saud | Shervin and Amir |
| Experiments | All |  |

# References

1. Bogdanov, Alexander. “Optimal control of a double inverted pendulum on a cart.” Oregon Health and Science University, Tech. Rep. CSE-04-006, OGI School of Science and Engineering, Beaverton, OR (2004).
2. Yadav, Sandeep Kumar, Sachin Sharma, and Narinder Singh. “Optimal control of double inverted pendulum using LQR controller.” International Journal of Advanced Research in Computer Science and Software Engineering 2, no. 2 (2012).
3. C. Su, T. Wen, G. Huard, “Mechatronics MECH 474 Project Manual”, 2015.
4. Oskar Stattin, “Optimal Control of Inverted Pendulum,”' B.S. thesis, Dept. Math., Royal Institute of Technology Univ., Stockholm, Sweden, 2015.
5. Alexander Bogdanov, “Optimal Control of a Double Inverted Pendulum on a Cart”, OGI School of Science & Engineering, Hillsboro, OR, Tech. Rep. CSE-04-006, 2004.
6. Steven A. Frank, “Control Theory Tutorial: Basic Concepts Illustrated by Software Examples”, SpringerBriefs in Applied Sciences and Technology, Chapter 11, pp. 85-89, 2018.

**Appendix A**

clc

clear all

% Load data from workspace

load('doublePend.mat');

clear 'dGama'; clear 'ltq';

Rm = 2.6;

Kt = 0.00767;

rmp = 6.35e-3;

tau = V \* Kt/(Rm\*rmp);

g=9.81;

theta2\_f = theta1\_f + theta2\_f;

% Define coeficient array

Gama = [0.5\*x\_dot\_c.^2,...

cos(theta1\_f).\*(x\_dot\_c.\*theta1\_dot),...

cos(theta2\_f).\*(x\_dot\_c.\*theta2\_dot),...

0.5\*theta1\_dot.^2,...

cos(theta1\_f-theta2\_f).\*theta1\_dot.\*theta2\_dot,...

0.5\*theta2\_dot,...

cos(theta1\_f),...

cos(theta2\_f)];

wm = tau .\* x\_dot\_c;

step = 10;

for i = 1:(length(Gama) - step):2

dGama(i,:) = Gama(i+step,:) - Gama(i,:);

ltq(i,1) = trapz(time(i:i+step,1),wm(i:i+step,1));

end

% Define lower bound for optimization

lb = [ 0.5 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ];

% Define upper bound for optimization

ub = [ 2 ; 0.1 ; 0.1 ; 0.01 ; 0.01 ; 0.01 ; 0.5 ; 0.5 ];

% Use lsqlin to find parameters

options = optimoptions('lsqlin','PrecondBandWidth',inf);

phi = lsqlin(dGama,ltq,[],[],[],[],lb,ub,[],options)

dt= 0.05 ; starttime = 15/dt; endtime = 45/dt;

d1 = phi(1);

d2 = phi(2);

d3 = phi(3);

d4 = phi(4);

d5 = phi(5);

d6 = phi(6);

f1 = phi(7);

f2 = phi(8);

% Define matrices using identified params at zero point

t1 = 0; t2 = 0;

d0=[d1, d2\*cos(t1), d3\*cos(t2);

d2\*cos(t1), d4, d5\*cos(t1-t2);

d3\*cos(t2), d5\*cos(t1-t2), d6];

dg=[0,0,0;0,-f1,0;0,0,-f2];

% Define the State Space System

A=[zeros(3),eye(3);-inv(d0)\*dg,zeros(3)];

B=[zeros(3,1);inv(d0)\*[1;0;0]];

C=eye(6); D=zeros(6,1);

sys=ss(a,b,eye(6),0);

Q =diag([5 50 50 20 700 700]); R=1;

% Compute the feedback k

k=lqr(sys,Q,R);

% Define the new system

sysnew=ss(a-b\*k,b,c,0);

% simulation and comparing data in plot

[y,t,xi]=lsim(sysnew,V(starttime:endtime),time(starttime:endtime));

figure

Simulated = iddata(y(:,1:3),V(starttime:endtime),0.05);

Measured = iddata([-

xc(starttime:endtime),theta1\_f(starttime:endtime),theta2\_f(starttime:endtime

)],V(starttime:endtime),0.05);

compare(Simulated,Measured)

figure

Simulated = iddata(y(:,4:6),V(starttime:endtime),0.05);

Measured = iddata([-

x\_dot\_c(starttime:endtime),theta1\_dot(starttime:endtime),theta2\_dot(starttim

e:endtime)],V(starttime:endtime),0.05);

compare(Simulated,Measured)

err\_beta=[norm(xc(starttime:endtime)),

norm(theta1\_f(starttime:endtime)),

norm(theta2\_f(starttime:endtime)),

norm(x\_dot\_c(starttime:endtime)),

norm(theta1\_dot(starttime:endtime)),

norm(theta2\_dot(starttime:endtime))];

% error estimation for each state

SE = err\_beta/size(y,1);